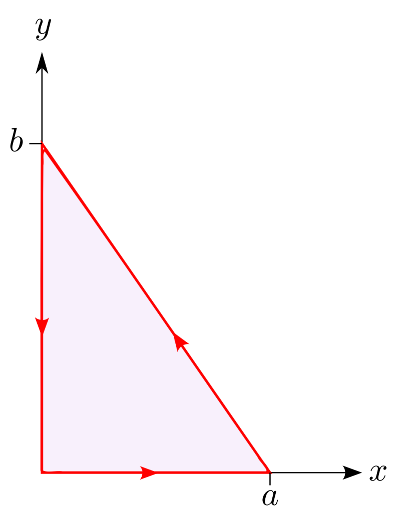
# PHYS 301 – Assignment #1

Due Wednesday, Oct. 2 at 14:00

1. Check the divergence theorem for the function $\mathbf{v} = 2r^3\,\hat{r}$. For your volume, use a sphere of radius $R$ centred at the origin. Confirm that the volume and surface integrals give the same result.

2. Check the divergence theorem for the function $\mathbf{v} = r^2\cos\theta\,\hat{r} + r^2\cos\phi\,\hat{\theta} - r^2\cos\theta\sin\phi\,\hat{\phi}$. For your volume, use a hemisphere of radius $R$. The flat surface of the hemisphere lies in the $xy$-plane and the curved surface is above the $xy$-plane ($z > 0$). Confirm that the volume and surface integrals give the same result.

3. Check Stoke's theorem using the function $\mathbf{v} = ay\,\hat{x} + bx\,\hat{y}$, where $a$ and $b$ are constants. Use the triangular surface and the path shown in the figure below. Confirm that the surface and line integrals give the same result.



4(a) Show that:

$$\int \delta(kx)\,\mathrm{d}x = \int \frac{1}{|k|}\delta(x)\,\mathrm{d}x,$$

such that:

$$\delta(kx) = \frac{1}{|k|}\delta(x).$$

(b) Use the result of (a) to evaluate:

$$I = \int_{-\infty}^{\infty} (5x+1)\,\delta\left[4\,(x-2)\right]\mathrm{d}x.$$

5. In Cartesian coordinates, the gradient of a scalar function is given by:

$$\nabla T = \frac{\partial T}{\partial x}\,\hat{x} + \frac{\partial T}{\partial y}\,\hat{y} + \frac{\partial T}{\partial z}\,\hat{z}.$$

If $T$ is now expressed in spherical coordinates (i.e. a function of $r$, $\theta$, and $\phi$), by the chain rule the derivatives in the gradient become:

$$\begin{aligned}
\frac{\partial T}{\partial x} &= \frac{\partial r}{\partial x}\frac{\partial T}{\partial r} + \frac{\partial \theta}{\partial x}\frac{\partial T}{\partial \theta} + \frac{\partial \phi}{\partial x}\frac{\partial T}{\partial \phi},\\
\frac{\partial T}{\partial y} &= \frac{\partial r}{\partial y}\frac{\partial T}{\partial r} + \frac{\partial \theta}{\partial y}\frac{\partial T}{\partial \theta} + \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial \phi},\\
\frac{\partial T}{\partial z} &= \frac{\partial r}{\partial z}\frac{\partial T}{\partial r} + \frac{\partial \theta}{\partial z}\frac{\partial T}{\partial \theta} + \frac{\partial \phi}{\partial z}\frac{\partial T}{\partial \phi}.
\end{aligned}$$

In class, we showed that:

$$\begin{aligned}
\frac{\partial r}{\partial x} &= \sin\theta\cos\phi,\\
\frac{\partial \theta}{\partial x} &= \frac{\cos\theta\cos\phi}{r}.
\end{aligned} \tag{1}$$

(a) Show that:

$$\frac{\partial \phi}{\partial x} = -\frac{\sin\phi}{r\sin\theta}$$

such that:

$$\frac{\partial T}{\partial x} = \sin\theta\cos\phi\frac{\partial T}{\partial r} + \frac{\cos\theta\cos\phi}{r}\frac{\partial T}{\partial \theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}$$

(b) Given that:

$$\begin{aligned}
\frac{\partial T}{\partial y} &= \sin\theta\sin\phi\frac{\partial T}{\partial r} + \frac{\cos\theta\sin\phi}{r}\frac{\partial T}{\partial \theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}, \\
\frac{\partial T}{\partial z} &= \cos\theta\frac{\partial T}{\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta},
\end{aligned}$$

show that, in spherical coordinates, the gradient operator becomes:

$$\nabla T = \frac{\partial T}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\,\hat{\phi}.$$